Vectors Linear Algebra X. Du

- Are basically matrices with either one row (row vector) or one column (column vector).
- Have **magnitude** and **direction**. Contrast to scalar quantities.
- **Magnitude**: $|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2}$ Pythagorean Theorem
  - **Unit vectors**: have magnitude of 1
  - **Standard basis unit vectors** in three-space: $\hat{i} = \langle 1, 0, 0\rangle$, $\hat{j} = \langle 0, 1, 0\rangle$, $\hat{k} = \langle 0, 0, 1\rangle$
- **Zero Vector**: vector of all 0's. $\vec{0} = \langle 0, 0, \ldots, 0\rangle$.
- Vector arithmetic: Let $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$. $\vec{a} + \vec{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}$ $c\vec{a} = \begin{pmatrix} ca_1 \\ ca_2 \\ \vdots \\ ca_n \end{pmatrix}$
- **Dot product**:
  - $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 + \ldots + a_nb_n$
  - Scalar!
  - $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ Proof with Law of Cosines
  - **Cauchy-Schwarz Inequality**: $|\vec{a}\cdot\vec{b}| \le |\vec{a}||\vec{b}|$
- **Cross product**:
  - Only valid in three-space. Let $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.
  - $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$
  - Vector! Direction - use right hand rule.
  - $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$
- $\vec{a}$ and $\vec{b}$ are perpendicular iff $\vec{a}\cdot\vec{b} = 0$. $\vec{a}$ and $\vec{b}$ are parallel iff $\vec{a}\times\vec{b} = \vec{0}$

Further notes:

- **Tensors**: extension of vectors. Scalars are 0-tensors (no direction), and vectors are 1-tensors (1 direction). An *n*-tensor has *n* directions.